**Lecture #016**

**POISSON PROCESS**

Recall that we have already studied the discrete RV with Poisson probability distribution. Now we shall study the corresponding random process. The two concepts are like two sides of a coin.

**Definition** Let X(t) represent the number of occurrences of a certain event in time interval (0,t). The discrete random process { X(t) } is called a Poisson process if the following postulates are satisfied:

1. Prob[ 1 occurrence in (t,t+Dt) ] = lDt + O(Dt2)

2. Prob[ 0 occurrence in (t,t+Dt) ] = 1 - lDt + O(Dt2)

3. Prob[ 2 or more occurrences in (t,t+Dt) ] = O(Dt2)

4. X(t) is independent of the number of occurrences of the event in any time interval before or after (0,t).

5. The probability that the event occurs a certain number of times in time interval (t0,t0+t) depends on t, but not on t0.

Derivation of the probability law:

Let l represent the average number of occurrences of the event in unit time, and let Pn(t) = Prob[ X(t) = n ].

From the postulates, and this, we can deduce that:

Pn(t+Dt) = Pn-1(t)lDt + Pn(t)[1- lDt]

🡪 How? Think about the two ways in which X(t) = n can come about in the time interval between t and t+Dt.

Therefore:

[Pn(t+Dt) - Pn(t)]/Dt = l[ Pn-1(t) - Pn(t) ]

Taking the limit as Dt 🡪 0, we get the differential equation:

d/dt Pn(t) = l[ Pn-1(t) - Pn(t) ]

Assume that the solution is of the following form, for some f(t):

Pn(t) = f(t)(lt)n/n!

When this is substituted in the differential equation, f(t) is easily found to be e-lt. Therefore from this we get the Poisson distribution which we have studied earlier.

Now review the five postulates with which we started.

Simple examples (from reference #2):

1. Customers arrive at a bank at an average rate of 3 per minute, according to a Poisson process. Find the probability that, in a 2 minute interval:

(a) exactly 4 customers arrive, and

(b) more than 4 customers arrive.

(a) Prob[ X(2) = 4 ] = e-664/4! = 0.133

(b) Prob[ X(2) > 4 ] = 1 - Sk=0,4 Prob[ X(2) = k ] = 1 - Sk=0,4 e-66k/k!

Each of the probabilities inside the summation is to be calculated exactly as is done in part (a).

The final answer is 0.715. [Verify!]

